11. Relativity

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The Nature of Time

What is time? Time moves. But what does *that* mean? Does time ever slow down? And if it did, would we notice? Could we notice? What is it that sets the "pace" of time? Nothing about time is obvious. Is time a universal concept that exists throughout the Universe? Can we go back in time? Can time run backwards? What determines the direction of time?

I can't answer all of those questions. Yet, given the mysterious nature of time, you may be surprised at some of the things we do know about it. Here is the strangest of them all:

If two twins are exactly the same age and one travels while the other stays at home, then when they are brought back together, the traveler will have experienced less time than the other twin.

There is nothing odder about time than that. Yet Einstein gave us a formula that tells us precisely how much less time the moving twin experienced. And that fact has been experimentally verified with very sensitive clocks flown on airplanes. Even radioactive atoms, when they move, experience less time than those that are stationary. That fact is verified every day at accelerator laboratories where such atoms are sent near the speed of light, and physicists note that their radioactive decays slow down.

The nature of time (and space) is at the heart of the theory of relativity. That's what this chapter is about. The theory of relativity was created by Einstein in the early 1900s. It consists of two parts. The first part is called "the special theory of relativity" and it has to do with the nature of time, space, energy, and momentum. It was in this work, published in 1905, that Einstein presented his famous equation $E = mc^2$. The second part was published in 1916 and is called "the general theory of relativity." It is really a theory of gravity. It "explains" all of gravity as due to a bending of space and time. This theory is needed to understand some of the recent discoveries in cosmology about the nature of the Universe. Most of this chapter will be about special relativity.

This chapter departs a bit from my previous philosophy. Future presidents don't really need to know about the theory of relativity. It is important, however, to physicists, to philosophers, to those who plan trips to other planets, and to anyone who wants to have their mind stretched beyond what this course has already done.

The Fourth Dimension

We return to the question, "What is time?" The trivial answer that you will hear from some people is that time is "the fourth dimension." This sententious statement means less than it seems.

In this answer, the word "dimension" is being used in a very technical and narrow way. If you want to specify a *location*, you have to give three dimensions. They can be latitude, longitude, and altitude, or they can be x, y, and z. But given a coordinate system, three numbers are all you need to say precisely where an object is in that coordinate system.

If you want to specify an *event*, then it is sufficient to give the location and the time of the event. Suppose I were to tell you that there is an event at my house at 8 pm tonight. Then there is no confusion; you might not know what is going to happen, but you have located it in both time and space. The event can have a name, such as "Rosemary's birthday party" or "Muller goes to bed." But to be unique (Rosemary has a birthday each year, and Muller goes to bed almost every night) you also specify the time. This is what makes time the fourth dimension, although in a trivial way. It is only the forth dimension because you are specifying an event rather than just a location.

That is not what is interesting about time. What is interesting is that the amount of time can change depending on the velocity that an object is moving. That idea requires some explanation.

Time Dilation

I described in the opening of this chapter how two twins can experience different amounts of time. That seems to violate common sense. How can it be true? The answer is that the effect is very small unless the velocity is very fast--that's why you never notice it. In fact, it is difficult even to measure the effect unless the velocity is near the speed of light. For airplanes moving at 675 mi/h, the effect is about 5 $\times 10^{-13}$. That means that if you traveled at this speed for one day, you would lose 44 nanoseconds. (That is the time it would take light to travel 44 ft.) If you fly for a *year*, you will experience 16 microseconds less time than your twin who doesn't travel.

This "small" effect becomes large if the velocity approaches the speed of light. Suppose one twin, John, is stationary while the second twin, Mary, moves extremely fast at 60% of the speed of light. In science fiction parlance, that would be called "light speed 0.6." In physics, we usually call it "beta," or use the Greek letter β (looks like a B with a tail).

$$\beta = v/c = \text{light speed}$$

Let me now show you how to do the slow down calculation yourself. Einstein gave an exact formula for calculating this. Time will slow down by a factor f = sqrt($1-\beta^2$). I don't require that you memorize this formula, but you might want to anyway, because then you can do real relativistic calculations.

Let's get back to our example. If Mary's light speed $\beta = 0.6$, then the equation gives f = sqrt(1-β^2) = sqrt(1-0.36) = sqrt(0.64) = 0.8.

So Mary's time will go slower, at a rate that is only 0.8 that of John's time. So if John ages 1 year, then Mary will age only 0.8 years. When they compare ages, John will be 0.2 years older than Mary (i.e. a little more than 2 months older). Yet they are twins, born at the same time.

The effect gets much more dramatic as Mary's velocity increases. Suppose she travels at light speed 0.99999, i.e. at 99.999% the speed of light. If you plug that into the time equation, you'll find Mary's time progresses at a rate only 0.0045 the rate of John's time. If John ages 1 year, Mary will age 0.0045 years. That is only 0.0045 x 365 days = 1.6 days.

Not only that, but she will *experience* only 1.6 days, while John experiences a full year. If they began as freshmen, Mary will still be a freshman, but John will be a sophomore.

The fastest that any astronaut has ever traveled is approximately Earth escape velocity, about 11 km/s. This is equivalent to light speed $\beta = 0.0037$. Plug this into the time equation (use a calculator) and you'll find that the astronaut time goes at a rate that is 0.99999933 slower than Earth time. That isn't a big change (since the number is so close to 1). If he travels for 1 year (that is, 365 days x 24 h x 3600 s/h = 3.16E7 s), then he will experience 0.02 s less than if he stayed at home. That's not enough for him to notice unless he is carrying a very accurate clock.

We have sent radioactive atoms at velocities close to the speed of light, and their radioactivity does slow down, by exactly the predicted factor.

Suppose we go faster than the speed of light, for example, we try light speed $\beta = 2$. Try plugging it into the equation and see what happens. We'll discuss faster-than-light particles later in this chapter.

The Einstein Factor γ

In addition to f, we often find it useful to use the function "gamma" (Greek letter γ, which looks like a little fish swimming straight down), also called the "Einstein factor." Gamma is just $1/f$. Once you calculate the Einstein factor, you know most of the things that the theory of relativity predicts.

$$\text{gamma} = \gamma = 1/f = \text{Einstein factor}$$

Here is a list of the values of f and γ that you get for different values of v/c:

v/c	0	0.25	0.5	3/5	4/5	0.9	12/13	0.99	0.999	0.99999	1
f	1	0.97	0.87	4/5	3/5	0.44	5/13	0.14	0.045	0.0045	0
γ	1	1.03	1.15	5/4	5/3	2.3	13/5	7.1	22.4	224	infinity

Optional for those who like to play with math: Maybe you noticed that some of the values in the table are given as exact fractions, rather than in terms of their decimal approximations. You can verify that if you have an exact right triangle, e.g. a 3:4:5 triangle, or a 5:12:13 triangle, then values of velocity given by fractions made of these numbers will give exact fractions for the time factor (f) and the Einstein factor (γ).

But … but … how can time depend on velocity?

How can time depend on velocity? That sounds absurd. It goes against intuition. It goes against everything we experience.

Or does it? Why do you believe that time is independent of your path? Did you always believe it? I'll bet that you didn't believe it when you were a child. An hour at the dentist's office didn't seem to go as fast as an hour in a swimming pool. But you were trained to watch clocks and to "be on time," and you finally learned that there is a "universal" time that you can follow in order to get to appointments on time. But it was never intuitively obvious.

Nor is it true. It is *almost* true, however, since for everyday velocities, the gamma function is very close to having a value of 1, the value for which there is no time dilation. Even in the airplane example, the factor f was very close to the value of 1. At 675 mph, v/c is 10^{-6}. The factor $(v/c)^2$ is 10^{-12}. The time factor f, on most calculators, will come out to be exactly 1, since they don't handle enough decimal places! If calculated on a sufficiently accurate computer, the factor $f = 0.9999999999995$. (There should be 12 nines in that number, if I entered it correctly.) It can be written as $1 - 5 \times 10^{-13}$.

That is pretty close to 1, so it is hard to notice the difference.[1] Nevertheless, in 1972, two scientists recognized that clocks had become accurate enough that the twin effect could be measured even in an airplane. Their results were published in *Science Magazine*.[2]

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[1] Here is a way to do the calculation without a computer. This is only for those who love math: when v is very small, we can write $\mathrm{sqrt}[1 - (v/c)^2] \approx 1 - (v/c)^2/2$. (To prove that, multiply $1 - (v/c)^2/2$ by itself, and ignore all terms that have $(v/c)^4$ in them.) In our example, $(v/c)^2 = 8 \times 10^{-13}$. So $f = 1 - 4 \times 10^{-13}$. That is the number I gave in the text.

[2] I don't expect you to read this, but here is the reference: J.C. Hafele and R.E. Keating, "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains," *Science* 177 (1972): 166.

Their results confirmed that even at the "slow" velocity of airplanes, the equation works. The "moving" clock experienced less time than the stationary clock on the ground.[3]

Not all motion is relative

Isn't all motion relative? Who is to say which clock is moving? I raise this issue only because it is a favorite complaint made by people who have studied a little bit of relativity theory. In fact, in relativity theory, it is *not* true that all motion is relative. The clock that is moving is the one that had to be accelerated to make it return home.[4]

Optional: Here are some details, for those interested. The twin equation is good only if applied in a single "inertial" frame of reference, i.e. one in which there is no acceleration without applied forces. The frame of the stay-at-home twin can be used. The frame of the traveling twin cannot be used, because that twin must change direction halfway through the trip. Which twin changed velocity? That is unambiguous, since it takes force to accelerate.

If you are willing now to believe that time depends on velocity (or at least accept the concept for a while), then you are ready to move on to a few more of the astonishing facts of relativity theory. You need no more math. All the results use the same functions, f and γ.

Length Contraction

When an object moves to a velocity v, it gets shorter. Its new length is the old length multiplied by the factor f. This effect is called the "Lorentz contraction." It is named after the person who first proposed it, even before Einstein. (Einstein published his theory of relativity in 1905. But it was based on two decades of work preceding it, including that of H. A. Lorentz.) Just to make it explicit, if a stationary object has rest length L_S, then when the object moves, its new length is L_M given by:

$$L_M = f\, L_S$$

The moving object is shorter, since its length is the rest length multiplied by f.
This contraction turns out to be tricky to measure. If your ruler is moving with you, then it shortens too, so you think your object hasn't gotten any shorter! To see the effect, you have to have a stationary ruler measuring a moving object.

[3] In their work, Haefele and Keating had to calculate the gravity effect, something we haven't discussed yet, as well as the twin effect. Their result matched the prediction for the combined effects.

[4] I've written a more detailed article on this. It was published in the *American Journal of Physics* Vol. 40, pp. 966-969.

Remember, the factor f is so close to 1 at everyday speeds, that this effect is difficult to notice. It becomes really important only when v approaches c, which occurs for particles emitted from radioactive nuclei, in accelerators (popularly called "atom smashers"), and in cosmology where distant galaxies are moving away from us at speeds that approach the speed of light.

Velocities Don't Just Add

Suppose you are moving at half the speed of light, $c/2$, and you crash into someone coming from the opposite direction, who is also moving at half the speed of light. What is your relative velocity? You would probably expect it to be c, the sum of the two velocities. Suppose you were moving at .75 c, and so was the other person. Was your relative velocity 1.5 c?

Here comes another surprise: the answer is no. The correct answer was calculated by Einstein, taking into account both the space and time changes. He showed that the relative velocity is given by:

$$V = (v_1 + v_2)/(1 + (v_1\, v_2/c^2)$$

You are not required to know this formula, but the consequences are important. Watch what happens when you put $v_1 = 0.5\ c$ and $v_2 = 0.5\ c$. You get

$$\begin{aligned} V &= (c/2 + c/2)/(1 + c/2\ c/2\ /c^2) \\ &= c/(1 + 1/4) \\ &= 0.8\ c \end{aligned}$$

So the relative velocity is less than c, i.e. it is less than the speed of light. Suppose we try $v_1 = 0.9\ c$ and $v_2 = 0.9\ c$. Then the relative velocity is still less than c! Try it yourself. The answer I get is $V = 0.994\ c$. In fact, no matter how close the individual velocities are to c, their relative velocity is still less than c. (If you like math, you might enjoy trying to prove that.)

One consequence of this is that you cannot get velocities equal to or greater than c by adding together smaller velocities. For example, if you have a multistage rocket, with the first stage getting to 0.9 c, and the second stage getting to 0.9 c relative to the first, the total velocity that you will get for the second stage is only 0.994 c.

Invariance of the speed of light

Suppose a photon is coming towards you, traveling at the speed of light c. You move with a velocity v towards that photon. With what velocity will the photon hit you when you are moving? It is natural to assume it will be greater than c, but that is not true. I'll use the velocity equation, with my velocity $v_1 = v$, and the photon velocity $v_2 = c$. (If this confuses you, skip to the last paragraph in this section.) This gives the new photon velocity as follows:

$$
\begin{aligned}
V &= (v_1 + v_2)/(1 + (v_1\, v_2/c^2) \\
&= (v + c)/(1 + v\, c/c^2) \\
&= (v + c)/(1 + v/c)
\end{aligned}
$$

Now I multiply the numerator and denominator by c:

$$
\begin{aligned}
V &= c(v + c)/c(1 + v/c) \\
&= c(v + c)/(v + c) \\
&= c
\end{aligned}
$$

(This is the place you were allowed to skip to.)

So the photon hits me with velocity c. Even though I am approaching it, it still hits me with the same speed c. This surprising property is sometimes called "the invariance of the speed of light." In fact, in many books it is taken as a fundamental assumption, and then all the rest of relativity theory can be derived from it.

Energy and Mass

Einstein noticed another consequence of the velocity equation. The old concept of momentum conservation no longer worked. If two objects have equal and opposite momentum (mass times velocity; see Chapter 3), then on a collision the result will be at rest. Momentum is conserved. But when Einstein calculated the momentum for objects in which both were moving, that was no longer true. Einstein guessed (correctly, it turns out) that momentum conservation was correct, and the velocity equation was correct. The mistake was that the mass of an object is not really constant, but that it depends on velocity. If we let m_0 be the mass of the object when it is at rest, then its mass when it is moving, its "kinetic mass," is

$$
\begin{aligned}
m &= \gamma\, m_0 \\
&= m_0/f
\end{aligned}
$$

So kinetic mass is *bigger* than rest mass by the same factor that tells us how much length and time are *smaller*. For mass, we divide by f rather than multiplying.

When something gets a bigger mass, that means that it gets harder and harder to accelerate, and it also means that the pull of gravity will increase. But it also has an important consequence for energy.

E = mc²

Einstein took these calculations one step farther and calculated the energy of a moving object. He deduced that it is given by:

$$E = \gamma\, m_0\, c^2$$

If I restate this by defining the ***kinetic mass***

$$\text{kinetic mass} = m = \gamma\, m_0$$

then this equation becomes the most famous equation in all of relativity theory:

$$E = m\, c^2$$

At first look, it appears that the energy of an object does not depend on velocity! But that isn't correct, because the mass depends on velocity. (The mass $m = m_0/f = \gamma\, m_0$.) Nevertheless, the equation looks very different from the old kinetic energy equation:

$$E = (1/2)\, m_0\, v^2$$

How can the two equations be reconciled? At first look, it seems impossible. At zero velocity, the kinetic energy equation gives $E = 0$, whereas the Einstein's equation gives $E = m_0c^2$. Those are very different numbers. Because the value of c is huge, the zero-velocity energy m_0c^2 is very huge.

Even though they disagree, the two equations are more similar than you might think. For small velocities, it is possible to show that if the velocity is low, then the Einstein equation can be approximated as follows:

$$E \approx m_0\, c^2 + (1/2)\, m_0\, v^2$$

(If you are mathematically inclined, you might try to prove this. I give some hints in a footnote.[5])

This approximate version of Einstein's energy equation, valid only at low velocities, says that the energy is equal to the old term ($1/2\, mv^2$) plus a new term that is constant: $m_0\, c^2$. This constant term has a famous name. It is called the *rest energy*. The smaller part, the $1/2\, mv^2$ term, is still called the *kinetic energy*. We now say that the total energy is the sum of the rest energy plus the kinetic energy.

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[5] If you want to try deriving this, here are the key equations you will need. Assume that $(v/c) = \beta$ is a very small number. There are two key approximations you must make. The first is that sqrt($1 - \beta^2$) $\approx 1 - \beta^2/2$. You can check this equation on a calculator by putting in some small numbers (e.g. 0.01) for β. The second approximate equation is $1/(1 - \beta^2/2) \approx 1 + \beta^2/2$. Check this one too. Both of these approximations can be derived using algebra, calculus is not required. You can check the first equation by squaring it; you can check the second by cross-multiplication; in both cases, throw away the tiny β^4 terms.

This equation says that even a particle that is at rest stores enormous energy, roughly the same energy it would have (by the previous equation) if it were moving near the speed of light. But how do you extract this energy? Part of it is extracted when we have a radioactive decay. The mass of the debris is less than the mass of the original particle, since some of the energy has been turned into energy.ω But in typical chemical changes, the mass doesn't change. In chemistry this is called "conservation of mass." Since the mass is constant, you can ignore it when looking at energy conservation. That's why it was never noticed.

Turning mass to energy, and energy to mass

With the discovery of radioactivity, the presence of the relationship between mass and energy has been measured. When a radioactive particle decays, the sum of the masses of the pieces is less than the mass of the original atom. The energy of the radioactive explosion came from converting mass to kinetic energy.

How much rest energy is there in a particle? For a proton, the rest energy is 938 MeV. That is huge compared to the typical 1 MeV released in radioactive decay. But notice that the rest energy of the electron is not huge, only 0.511 MeV. That is why electrons can be *created* in beta decay. (Recall the analogy I make in Chapter 4: the electron is not inside the nucleus, any more than a sound wave is inside your body waiting for you to speak; both are created at the moment they "come out.")

Some particles have all of their energy in kinetic energy. An example is the photon. All of the energy of a photon can be absorbed when that photon hits an object.

Physicists usually say that the "rest mass" of a photon is zero. That is an odd statement, since you can't bring a photon to rest. But if you take energy away from a photon (perhaps by scattering it off an electron) then the energy can be made to get smaller and smaller. Eventually the photon has energy approaching zero, and that could only be true if it had no rest energy, i.e. had zero rest mass.

Antimatter engines

Could we release the rest energy of the electron, and turn it into kinetic energy? Yes, there is one way: use antimatter. An anti-electron, also called a *positron*, has the same mass as the electron but opposite charge. Bring it together with an electron, and the charges will cancel, and all of their mass energy will be released. Two photons will emerge in equal and opposite directions, and all the energy of photons is kinetic (since they have zero rest mass, as we will see in the next section). Therefore all that energy can be turned into heat.

You can do much better if you bring a proton together with an antiproton. The process of releasing all this energy is called annihilation. When a proton is annihilated, virtually all 938 MeV of its rest energy is released. That's why antimatter drives are so popular in science fiction stories. Matter and antimatter may constitute the ultimate energy fuel.

Kinetic energy can also be turned into mass. When a gamma ray passes close to a nucleus, we often observe a phenomenon called "pair production." The energy of the gamma ray is suddenly converted into the mass of a particle and an antiparticle, such as an electron and positron. This is a fairly common occurrence for high energy gamma rays, and it is seen, for example, in cosmic radiation. The first positron ever detected was one that had been produced by a cosmic gamma ray. Other collisions (such as between electrons) can also create such pairs. The first antiproton ever detected was created as part of a proton-antiproton pair in the Berkeley atom smasher known as the Bevatron located at the Lawrence Berkeley Laboratory.

The Photon Has Zero Rest Mass

Another way to deduce that a photon has zero rest mass begins with the equation $E = mc^2 = \gamma\, m_0 c^2$, which is true for all particles. Remember that $\gamma = 1/f$. When the velocity $v = c$, then $f = 0$, and this makes γ infinite. This seems to say that any particle that travels at the speed of light, such as a photon, must have infinite energy! But, no, that result isn't right. The equation contains the rest mass m_0. The rest mass of the photon must be zero. That is the only number which, when you multiply it by infinity, gives a non-infinite answer. In fact, zero times infinity is "indeterminate." That means that it could be any number.[6] You can't tell what it is unless you have another equation. For the photon, we do have such an equation. From Chapter 10 "Quantum Physics" we have $E = h\,f$, where h is Planck's constant, and f is the photon frequency.

Likewise, we conclude that if a particle has mass = 0, but has energy, then it must move at the velocity c. It can't move faster, and it can't move slower. Its energy is not related to its velocity, but only to its wave frequency.

But what about light in glass? Doesn't that move at c/n, where n is the index of refraction. Isn't that slower than c?

The answer is that the equation for the energy did not include the glass; if you put that term in, then you no longer conclude that light must travel at c. But in a vacuum, a zero mass particle must move at c, all the time.

You might notice that the kinetic mass of a photon, $m = \gamma\, m_0$, is not zero. It is only the rest mass m_0 that is zero. That is odd, given the fact that the photon cannot be brought to rest! Nevertheless, the constant m_0 in the equation that defines the rest mass, $E = \gamma\, m_0 c^2$, must be zero for the photon.

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[6] One way to see that is the fact than any number x divided by zero is infinite: $\infty = x/0$. If you cross multiply, you get $0\,\infty = x$

The fact that the photon has nonzero kinetic mass suggests that it feels the force of gravity. In fact, Einstein's theory of gravity, called "general relativity," predicts that photons will be affected by gravity.

Massless Particles Have No Time

The time dilation factor for a massless particle is still f but, for such a particle, f is zero! That means while an hour passes for you and me, the time that passes for a massless particle is zero. (If this bothers you, then let's just assume that the particle has a very tiny mass.) It is moving at *almost* the speed of light, and f is very close to zero. So the particle experiences very little time.

If a massless particle does not experience time, can it undergo radioactive decay? Think about this for a moment. What would you guess? The answer is no. Massless particles, which have no internal time, cannot radioactively decay.[7]

Do neutrinos have mass?

The neutrino is a particle that we always thought had mass zero. Neutrinos are emitted in many nuclear decays, and they travel through most matter without being noticed. They are emitted from the sun, but sensitive experiments to detect them have shown that we detect only 1/3 of the number we expect. What is going on?

New experiments have been done that suggest an answer: the neutrinos are changing from ordinary neutrinos into exotic neutrinos called "muon neutrinos" and "tau neutrinos." (Ordinary neutrinos are often called "electron neutrinos," since they are never created by themselves but always in combination with electrons.)

But if the neutrinos are changing, they must be experiencing time. That means that they aren't traveling at the speed of light. That means that they aren't truly massless.
So by observing a low number of solar neutrinos, physicists were led to conclude that neutrinos must have some mass. That was a conclusion that surprised nearly everyone.

Why You Can't Get To Light Speed

Suppose you take a massive particle (such as an electron, or a person) that has a nonzero rest mass m_0, and you accelerate it to the speed of light. Then its energy is

$$E = \gamma\, m_0\, c^2 = \infty$$

[7] Even without invoking the "zero time" argument, you can prove they don't decay from the more formal approach, that is, by showing such a decay cannot simultaneously conserve energy and momentum.

For any particle that has nonzero rest mass (e.g. you), if you accelerate it to the speed of light c, then its energy will be infinite. That means that it would take infinite energy to reach c. That is the fundamental reason why you can't travel at light speed.

E = mc^2 and the atom bomb

Many people think that Einstein's discovery of the equation $E = m\,c^2$ led to the invention of the atomic bomb. In fact, this equation is irrelevant to the bomb. In the late 1800s, it was discovered that radioactive decay released a million times more energy than chemical explosions. That discovery was the key to nuclear energy. All that was really needed was the discovery of a suitable chain reaction, and that wasn't found until the late 1930s. Einstein's equation (which he published in 1905) showed that the enormous release of energy would be accompanied by a slight disappearance of mass. But nobody needed to know that in order to build an atomic bomb.

Beyond Light Speed: Tachyons

I showed that you can't send particles at the speed of light because that would take infinite energy. But can you send them faster? The surprising answer is maybe. If such particles exist, we call them *tachyons*. They have a surprising property: they must travel faster than light! They have the lowest energy when they move at infinite speed, and infinite energy when they approach light speed c.

Optional section: the math of tachyons

Look at the Einstein energy equation: $E = m\,c^2 = \gamma\,m_0\,c^2$. The Einstein factor γ is

$$\gamma = 1/f = 1/\mathrm{sqrt}[1 - (v/c)^2]$$

Note that γ goes to infinity when $v = c$. That means the particle would have infinite energy. But suppose v is greater than c. Then f is the square root of a negative number. That makes the energy factor γ imaginary, right? Right. Doesn't that make the energy E imaginary too?

The surprising answer is: not necessarily. Suppose there were a particle that had imaginary mass.

I know, now you are completely confused. Does a particle with imaginary mass mean that the particle doesn't really exist? Is it imaginary? If you've never felt comfortable with "imaginary" numbers, don't worry. Stick with me, and learn the conclusions, if not the logic that we use to get there.

The two imaginary numbers, the mass and f, when multiplied together, create a real number. That means that everything works out okay. We can re-write the energy equation in terms of real numbers. Let the mass of the tachyon be ($i\,m_0$), where i is the square root of –1, and m_0 is a real number. Then, for a tachyon, the two values of i will cancel, and we'll get the following:

$$E = m_0\, c^2/\mathrm{sqrt}[(v/c)^2 - 1]$$

This is the tachyon energy equation. Note that when v becomes infinite, the energy E goes to zero. And when v is equal to c, the energy E is infinite.

If you think that is weird, be assured that everyone else thinks so too. But that doesn't prevent it from being true. Of course, as of the writing of this book, no tachyons have been discovered, so they may not exist.

More About Tachyons

As I said, a tachyon is a particle (not yet discovered, although people have looked) that travels faster than the speed of light. But how can you get it going that fast? Doesn't it have to start at rest, and so it would take infinite energy to get just up to c and, therefore, it could never go beyond? No, tachyons are assumed to be *born* fast. Photons are never accelerated up to light speed, they start out that way. If tachyons can be created, when they are created they are already traveling faster than c. They can lose energy by speeding up, or gain energy by slowing down. (Note that is backwards from the way ordinary particles, such as electrons, behave.)

Tachyons may exist and, from time to time, physicists set up experiments to search for them. However, I do not think they will find the particles. The reason comes from another property of relativity having to do with the simultaneity of events.

Simultaneity

Different people can experience different amounts of time. That means there can be no such thing as a universal clock that tells everyone what time it is. Your time depends on your motion.

One consequence of this fact is that it is fundamentally impossible to determine whether two events are simultaneous.[8] And it gets even worse. The order in which two events happened can depend on the motion of the observer.[9] This happens with tachyons. Suppose you emit a tachyon at point A, and it travels to point B, where it is absorbed. To a different observer, one moving along the line between the two events with high speed,

[8] There is an exception. If the two events occur at the same location, then the order of events is unambiguous.

[9] To be precise, it depends on the velocity of the frame of reference, not on the velocity of the observer.

the order will be reversed. In that moving reference frame, event B will occur before event A.[10]

This fact may bother you more than anything else in this chapter. It means that, to one observer, event A precedes B, but to another observer, event B comes first. Suppose the tachyon was shot from a gun, and used in a murder. The shooter is arrested. In his defense he points out that, in a moving reference frame, the victim was killed before the gun was fired.

There is nothing in that story that violates the laws of physics. But it does violate our sense of free will. If the victim dies, can we still choose not to fire the gun? Physics doesn't answer this question. But this violation of causality is enough to cause many physicists to believe that tachyons probably do not exist.

The same can be said for time travel. Can you go "backwards" in time? If you could travel faster than the speed of light, then according to the equations of relativity, the order of events can reverse. Does that mean you can travel back in time? Well, the first problem is getting yourself to go faster than the speed of light. It takes infinite energy just to get up to *c*. But, could you tunnel across, go right to hyper light speed without ever being at light speed, by doing some sort of tunneling? Physics can't rule that out. But the problem, as with the tachyon gun, may be the havoc such events would play on our own concept of causality and free will. That is enough to get some physicists to guess that traveling backwards in time will turn out to be impossible.

Pondering Time

Having gotten this far in the theory of relativity, you have earned the right to speculate along with the best physicists. Just to get you started, here are some Questions to Ponder:

Is it possible that before about 14 billion years ago, when time began, there was no time? In fact, the statement "before time began" may be meaningless, since there was no "before" if there was no time.

Could time stop?

[10] The following math is only for those who have studied the Lorentz transformation in a different course. Let the two events be ($x1$, $t1$) and ($x2$, $t2$). In the moving frame, they occur spaced by a time interval $\Delta t' = \gamma(\Delta t - u\Delta x/c^2)$ where u is the relative velocities of the two frames. $\Delta t'$ will have a different sign from Δt only if $\Delta x/\Delta t > c^2/u > c$. That is possible if the two events have a tachyon moving from one to the other that has $\Delta x/\Delta t > c$.

What is the meaning of "now"? Can you explain it to someone? Can you write a paragraph explaining what it means? Does it mean the same thing to different people? Is my "now" the same as your "now"? What are you doing right now?

Why do we remember the past? Could the universe be such that we would remember the future instead? Or would we then call it the past? Could we remember some of the past, and some of the future--perhaps a 50/50 mix?

What is the "pace" of time? Is the rate of time set by our heartbeats and our rate of thinking? Is there any meaning to the "passage of time"? Does time "move"? If time sped up and slowed down, would we notice?

Space exists in three dimensions (at least). Could there be two dimensions of time? Could time run simultaneously in two separate ways? Or three? What would life be like with two dimensions of time?

Answers to the Questions

Actually, I don't know the answers to any of them. I have some speculation about remembering the past, rather than the future, but that is about it. Most of the questions I have given above are not considered part of physics, but I think that is only because we have made so little progress in understanding them.

END OF CHAPTER

Quick Review

An event can be specified by four dimensions: three positions, and the time of the event. But the time interval between two events depends on the frame of reference. The amount of time experienced by a moving traveler is less than that by someone who is stationary, by the factor

$$f = \text{sqrt}[1-(v/c)^2]$$

This difference in experienced time has been verified by experiments with clocks on airplanes and with accelerated radioactive particles. For low velocities, $f \approx 1$, and that is why we don't usually notice the effect. But as v gets close to c, the value of f can be much less than 1. The same factor also describes length contraction, also known as the Lorentz contraction. According to this result, objects get shorter along the direction of motion.

Velocities don't add in the usual way. No matter who observes light, it will appear to be moving at c. That is the "invariance of the speed of light." If the object is moving at a speed less than c, then it will still be moving at less than c for all observers, no matter how fast the observer is moving.

The energy of a moving object is given by the famous equation $E = m\,c^2$. This includes both rest energy and kinetic energy. The m in this equation is the kinetic mass, which grows at high velocity according to the equation $m = m_0/f = \gamma\, m_0$ (m_0 is a number called the "rest mass" that doesn't change.) At low velocities, the total energy is the rest energy plus $(1/2)mv^2$. Rest mass can be turned to kinetic energy. This is done in nuclear decay, and in annihilation. A photon has rest mass zero. Such particles do not experience time, and cannot undergo decay. We used to think neutrinos also have rest mass zero, but since they can change into other neutrinos, they must have some mass. Einstein's equation $E = m\, c^2$ was not important in the invention of the nuclear reactor or the atomic bomb. Tachyons are hypothesized particles that only travel at speeds faster than light. They reach zero energy at infinite velocity, and infinite energy when they travel at c.

Because of the variability of time in relativity theory, it is impossible to define "simultaneous" in an absolute way, unless two events occur at the same location.

Essay Questions

What usual concepts of time are upset by the theory of relativity? What do most people accept as obvious, that turns out not to be true?

Energy can be converted to mass, and mass to energy. Describe how. Give specific examples.

Physicists often say that people will never be able to travel at the speed of light. Explain why they believe that.

Describe the peculiar phenomenon of the "twin effect" in relativity.

Short Questions

Which of the following quantities do not depend on velocity?

( ) m
( ) m_0
( ) kinetic energy
( ) total energy

In the twin effect, the traveling twin is

( ) younger
( ) lighter
( ) longer
( ) older

Neutrinos are believed to have mass

( ) zero
( ) small, but not zero
( ) infinite
( ) imaginary

When a particle approaches the speed of light, its mass approaches

( ) zero
( ) infinity
( ) m_0

In everyday life, we don't see the effects of relativity because

( ) they only occur when near *c*
( ) they are too small to detect easily
( ) we have become accustomed to them, so we don't notice

Tachyons are particles that

( ) travel at *c*
( ) have zero mass
( ) travel faster than *c*
( ) have infinite energy

Rest-mass 0 and neutrinos

A particle really does travel at the speed of light. We conclude that (be careful, only one answer is correct)

( ) its energy is infinite
( ) it violates special relativity
( ) its energy is zero
( ) its rest mass is zero

The photon has a rest mass approximately equal to

( ) the mass of an electron
( ) 0.000003g
( ) 0
( ) 3.14 g

If Muller discovers a new, energetic particle (the "Mulleron") that has zero mass, then we can conclude that the Mulleron

( ) can pass directly through matter
( ) is a black hole
( ) travels at the speed of light
( ) is radioactive

Which of the following is not a zero-mass particle?

( ) neutrino
( ) graviton
( ) gamma ray
( ) beta ray

The speed of a massless particle (careful, may be a trick)

( ) is the same to all observers
( ) is faster to people moving in the same direction
( ) slower to people moving in the same direction

What observations show that neutrinos have mass?

( ) Neutrinos being emitted in nuclear decays.

( ) Neutrinos change from ordinary to exotic neutrinos meaning that they experience time.

( ) Neutrinos have been seen to travel at the speed of light.

( ) Physicists have not been able to prove that neutrinos have mass.

Neutrinos are believed to have mass

( ) zero
( ) small, but not zero
( ) infinite
( ) imaginary

Identify which are the particles that pass easily through the Earth.

( ) cathode rays
( ) x-rays
( ) neutrons
( ) neutrinos

The observation of neutrino oscillations proves

( ) neutrinos do have mass
( ) neutrinos violate relativity theory
( ) neutrinos have charge
( ) neutrinos travel at the speed of light

A neutrino can have which of the following values for its energy?
(Careful, this is a tricky question.)

( ) 0 joules
( ) 200 joules
( ) either of the above two values
( ) –200 joules (negative)
( ) any of the above three values

Length contraction/time dilation

A moving object is

( ) shorter and younger
( ) longer and older
( ) shorter and older
( ) longer and younger

In the twin effect, the traveling twin is

( ) younger
( ) lighter
( ) longer

A radioactive particle has a half life of 1 second. If it moves at 3/5 the speed of light, its new half life will be ___________________.

The Lorentz contraction refers to the fact that

( ) metals contract when they are cooled
( ) ice contracts when it melts
( ) plastics contract when they are cooled
( ) moving objects get shorter

The Lorentz contraction says that a football thrown past your face

( ) is shorter by a factor of gamma
( ) is longer by a factor of gamma
( ) is the same length as it is at rest
( ) appears different, but is not truly different.

If one person travels at 99% the speed of light for 7 years, how much younger will he be than his identical twin who didn't travel?

( ) 1 year
( ) 8 years
( ) he will be the same age
( ) he will be older

Jane and Lane synchronize their watches on Earth. Jane remains on Earth while Lane takes a round trip to Pluto at near light speed. When they get back together, whose clock is correct?

( ) Jane's clock (on Earth)
( ) Lane's clock (on the spaceship)
( ) both clocks are correct
( ) neither clock (since the Earth is always moving too).

An astronaut Alice makes a round trip at 3/5 the speed of light. Her twin Bob stays on Earth. When Alice returns, Bob is

( ) younger than Alice
( ) older than Alice
( ) the same age as Alice

Jim and Mary are both from Berkeley and are the exact same age. Jim travels at a velocity of 65 mi/h to Los Angeles and waits there. Mary drives the next day at 70 mph.
When she gets to Los Angeles, who is older?

( ) Jim
( ) Mary
( ) They are the same age
( ) It depends on the distance

Tachyons

According to Muller, tachyons

( ) have been produced by him
( ) are made only by large accelerators
( ) are theoretically impossible
( ) conflict with "free will"

Tachyons

( ) have never been observed
( ) were discovered in the last 10 years
( ) are used for medical imaging
( ) are the components of protons

Energy/traveling *c*

The most accurate equation for energy is
( ) $E = (1/2)\, m\, v^2$
( ) $E = m_0c^2 + (1/2)\, m\, v^2$
( ) $E = m\, c^2$
( ) $E = m_0c^2$

We measure that a particle travels at the speed of light. We conclude (be careful, only one answer is correct)
( ) its energy is infinite
( ) it violates special relativity
( ) its energy is zero
( ) its rest mass is zero

When an object's velocity approaches the speed of light, its energy approaches:
( ) $m\, c^2$, where m is the rest mass
( ) (1/2) $m\, c^2$, where m is the rest mass
( ) $m\, c^2 + (1/2)\, m\, c^2$ where m is the rest mass
( ) infinity

If you travel at 50% of the speed of light, your mass is increased by a factor of:
( ) 1.15
( ) 1.50
( ) 2
( ) 1

According to relativity theory, the energy of an object of mass m_0 that is moving at velocity v is
( ) 1/2 $m_0\, v^2$
( ) $m_0\, c^2$ (where c is the velocity of light)
( ) $\gamma\, m_0\, c^2$ (where $\gamma = 1/\surd(1 - v^2/c^2)$)
( ) $m_0\, c^2$

As the velocity of a moving object approaches the speed of light, what happens to its energy?
( ) It goes to zero
( ) It goes to infinity
( ) It becomes mc^2
( ) It becomes (1/2) $m\, c^2$

Electrons cannot go as fast as light because
( ) of the uncertainty principle
( ) it would violate free will
( ) they would expand to infinite size
( ) it would take infinite energy

How much energy would it take to send a small dust particle at the speed of light?

( ) energy from all the dynamite in the world
( ) energy from all of the nuclear weapons in the world
( ) the energy of a photon if you simply let it ride on a light photon
( ) all of the energy in the universe
( ) it is impossible

Miscellaneous

The effects of special relativity must be taken into account for

( ) semiconductor chips
( ) GPS
( ) lasers
( ) MRI

When a person moves at a velocity near the speed of light, which of the following are true--compared to a person at rest? (Check all that are applicable.)

( ) he doesn't age as rapidly
( ) his length gets longer
( ) his mass increases
( ) his energy increases

If the velocity is (12/13)*c*, then the gamma function is

( ) sqrt(5/13)
( ) 13/5
( ) 5/3
( ) none of the above

At very high speed, all of these change except

( ) length
( ) mass
( ) time
( ) electric charge

When the velocity of a proton is half of the speed of light, then the gamma function is approximately equal to

( ) 0
( ) 0.5
( ) 0.7
( ) 1
( ) 1.4
( ) 2

When a radioactive rock travels near the speed of light

( ) its mass increases and there are fewer decays
( ) its mass increases and there are more decays
( ) its mass decreases and there are fewer decays
( ) its mass decreases and there are more decay